A Performance Study of an Optical Burst Switched Network with Dynamic Simultaneous Link Possession *

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Abstract

In Optical Burst Switched (OBS) networks a burst may occupy a wavelength on one or more links as it travels through the network. In the literature, OBS networks have been analyzed assuming that each burst occupies only a wavelength on a single link. In this paper, we study analytically the performance of an OBS network with a mixture of different size bursts. The short bursts occupy a wavelength on a single link while the long bursts occupy *simultaneously* wavelengths on multiple consecutive links. We develop a queueing network, which models simultaneous link possession, and we calculate analytically the end-to-end burst loss probabilities over a path in the OBS network. Our results indicate that having a mix of various size bursts can greatly effect the burst loss probabilities in the network.

Keywords: Optical Burst Switching (OBS), optical networks, queueing network, simultaneous resource possesion, burst loss probability, decomposition algorithm, traffic model

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1 Introduction

Optical Burst Switching(OBS) is a promising solution for the all-optical WDM networks. Its dynamic nature leads to high network adaptability and scalability, which makes it quite suitable for the transmission of bursty traffic. Even though the term OBS is used to describe a variety of architectures and protocols [1], there are some definitive characteristics.

Most importantly, in OBS the data and the corresponding control information are transmitted separately in time and space through the network. The data in OBS comes from upper layer traffic which could be IP, SONET, ATM, Frame Relay or GbEthernet. The edge OBS end-devices collect this traffic, sort it per destination and assemble it into variable size data units, called *bursts*. For each burst, the OBS end-devices construct a *control packet*, which is transmitted an *offset* time prior to the transmission of the burst. As the control packet travels along the route, it is electronically processed and it reserves bandwidth resources for its corresponding burst. Upon receipt of the control packet, an OBS node schedules a free wavelength on the desired output port and configures its switching fabric to transparently switch the upcoming burst. The burst itself travels transparently as an optical signal from the source to the destination. This is advantageous because the OBS nodes just switch the optical signals without knowing the format or the transmission rate of the burst data.

In order to decrease the *end-to-end* transmission delay, most OBS architectures use a one-way signaling scheme. The transmitting end-devices do *not* wait for a positive acknowledgment from the destination end-devices that the control packet has been successful at reserving resources at each hop along the route. Instead, they wait the *offset* time and then transmit the burst. Therefore a burst may be dropped if it arrives at an OBS node, where the control packet was unsuccessful at reserving a wavelength on its desired output port. Therefore, the calculation of the burst loss probability is an important measure of the performance of an OBS network.

The more popular OBS protocols such as Just-Enough-Time(JET) [2], Just-In-Time (JIT) [3, 4] and Horizon [5] all use this one-way signaling scheme but employ different resource scheduling algorithms. The performance of these OBS protocols has been studied both through simulation and analytical models. The analytical studies mostly focus on a single output port of an OBS node and assume Poisson arrivals and full wavelength conversion [6, 7, 8]. Since these studies assume no buffers, the OBS output port is modeled as an M/G/W loss system, where W is the number of wavelengths, and the Erlang-B formula is used to calculate the burst loss probability. There are also studies of OBS with fiber delay lines [9, 7] or deflection routing [10, 11]. The optical composite burst switching (OCBS), where in case of contention only the initial part of the burst is dropped until a free wavelength becomes available, has been studied with ON-OFF arrivals [12] and Poisson arrivals [13]. OBS with QoS classes has been investigated in [14, 7] and an OBS architecture with centralized-signaling and guaranteed QoS classes is proposed in [15].

Note that all of the previously cited analytical models focus on a single OBS node. These models provide a limited insight about the overall performance of an OBS network. To our knowledge, the only published analytical model of an OBS network is the one in [16], where the OBS network is modeled by a network of loss nodes, each representing a link of W wavelengths.

Bursts are assumed to arrive in a Poisson distributed process and each burst occupies a single wavelength on each link along its source-destination path until it is lost or until it departs from the network. That is, each burst is at least as long as the source-destination path. This type of queueing network has been previously used to model teletraffic. It is analyzed by studying each node separately in an iterative fashion. The arrival rate of bursts to a node is modified to take into account blocking of bursts at previous links. This solution technique ignores the fact that the wavelength along the path are allocated and released dynamically. It has a good accuracy when the traffic load is extremely low [17].

In reality, however, a burst does not hold wavelengths on all links along its source-destination path. Bursts vary in size and the distance between two adjacent nodes also varies depending on the network's topology. In view of this, a burst may occupy *simultaneously* wavelengths on a variable number of links as it travels from its source to its destination. The number of links, simultaneously used by a burst, dynamically changes as the burst moves through the network. This behavior differs from the widely studied packet-switched or circuit switched networks. Therefore, new techniques have to be developed in order to investigate the performance of OBS networks, where a burst may simultaneously occupy different links as it moves through the network.

In general, depending on the maximum burst size, the minimum link lengths and the data rate, bursts may span one, two, three or even more links at the same time. In a long haul network, the bursts will most likely occupy only a single link at any time. However, in a metropolitan area network (MAN) the nodes are closer to each other and bursts may occupy more than one link a the same time. As shown by the study in [18], in an U.S. MAN of $\sim$3600 km^2 the core rings have a a circumference of 50-250 km and are made of several OXCs connected by short links. In one of the scenarios, the circumference of a core ring is only 37.5 km and it is made of 5 OXCs, which results in an average link length of $\sim$7.5km. Assuming that the data rate is OC192, in this example all bursts larger than $\sim$45KB will simultaneously hold resources on more than one link while bursts that are smaller than $\sim$45KB occupy resources on only one link at a time.

In this paper, we analytically study an OBS network with a mix of short and long bursts. Short bursts occupy a wavelength on one link at a time and long bursts occupy wavelengths on two consecutive links as they travel from their source to their destination. We propose a new queueing network which models *dynamic simultaneous link possesion*. In addition, we use a burst arrival process that is more realistic than Poisson. Our arrival process is called IDLE-SHORT-LONG and it accurately captures the burst transmission at the edge of the OBS network. The arrival process models the fact that a new burst can not arrive into the network until the previous burst is completely transmitted. In an earlier version of this work, we studied an OBS network, where all bursts occupied exactly two links [19].

The remainder of this paper is organized as follows. In Section 2, we describe the OBS network under study and in Section 3, we present a queueing network model of it and we describe the burst arrival process. In Section 4, we present a decomposition algorithm for analyzing this queueing network and in Section 5 we validate our algorithm against simulation

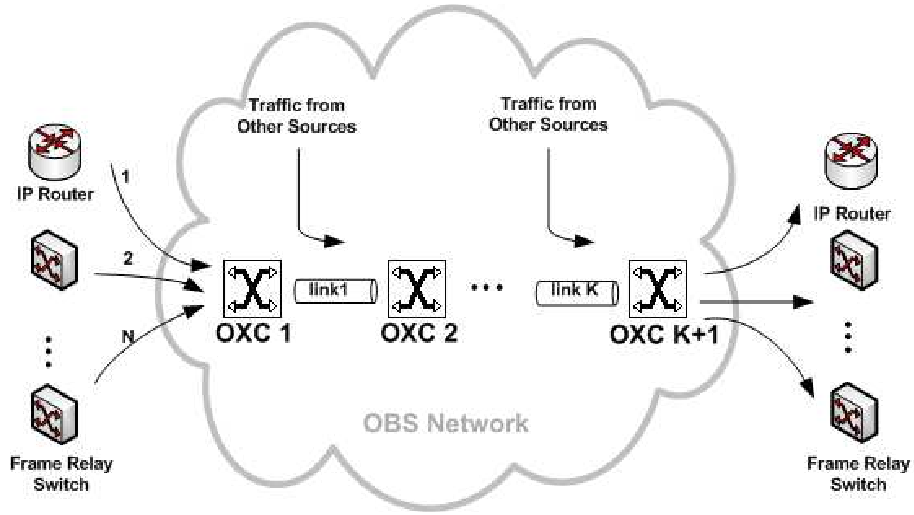

Fig. 1: A Path in an OBS Network

and discuss the results. We conclude this paper in Section 6.

2 The OBS Network Under Study

We analyze the performance of a specific transmission path in an OBS network. Therefore, we consider optical crossconnects (OXCs) connected in tandem, as shown in Figure 1. An OXC can optically switch a burst on an incoming wavelength of an input port to the same wavelength of any output port. Each OXC has full wavelength conversion capability, i.e., in the case of contention at an output port it can convert an optical signal from one wavelength to another. Two adjacent OXCs are linked by a single WDM link, which has W transmission wavelengths.

There are N transmitting OBS end-devices, linked to OXC 1, which transmit bursts to a number of OBS end-devices, linked to OXC K+1. Each transmitting OBS end-device is linked to OXC 1 by a single fiber and it may be equipped with one or more transmitters. There is zero burst loss on the link from OXC K+1 to each destination OBS end-device because they are linked via a "fat pipe". We will refer to the traffic generated from the N transmitting end-devices as the *cross traffic*. In addition, to the cross traffic we also consider traffic generated by any other sources in the OBS network. This traffic arrives at the intermediate links of the considered path. We refer to these arrivals as the *local* burst traffic. The *local traffic* also travels toward the same destination end-devices.

In this paper, we study an OBS network with a mix of *short* and *long* bursts. Short bursts occupy a wavelength on a single link while long bursts occupy wavelengths on two consecutive links *simultaneously*. The study presented in this paper, though limited to the case where bursts occupy either one or two links, provides a valuable first insight into the performance of an OBS network with dynamic simultaneous link possesion and the interdependence between different size bursts. We also note that our method of investigation can be modified to solve OBS networks with a mix of bursts that span any number of links at the same time.

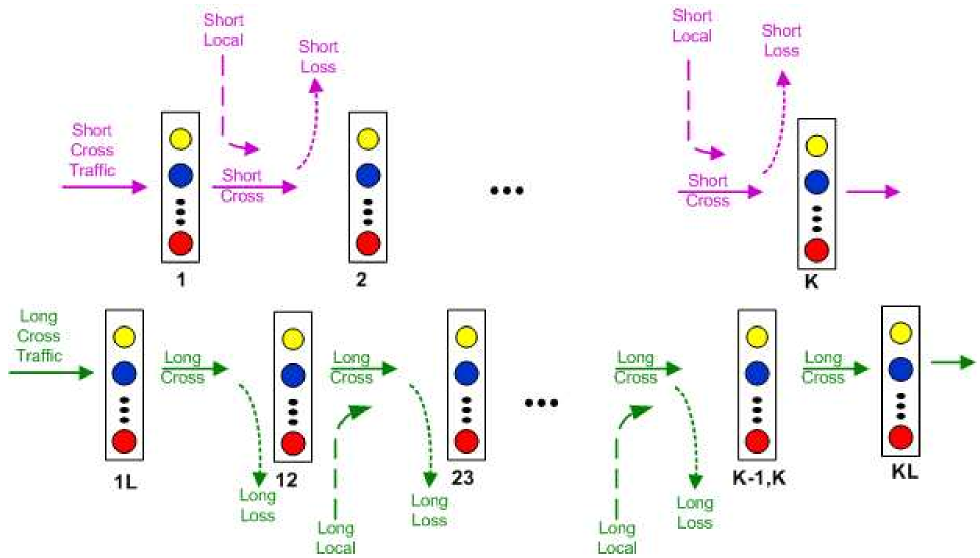

Fig. 2: The Queueing Network Model

3 The Queueing Network Model

The queueing model that we propose for an OBS path with a mix of short and long bursts is shown in Figure 2. Since there is no buffering in OBS, the queueing network does *not* contain any queues and therefore burst loss is possible at each link. The queueing network consists of a number of *loss* nodes with W servers each, where W is the number of wavelengths per link. Each loss node does not represent the state of a link but rather, as explained below, it represents the number of short bursts or long bursts that occupy one link or two successive links respectively.

3.1 Modeling Dynamic Simultaneous Possesion

The transmission of short bursts in the network is modeled by the loss nodes in the top row of Figure 2, i.e., node i, where $1 \leq i \leq K$. That is node i represents the number of short bursts currently holding a wavelength on link i. The transmission of the long bursts is modeled by the loss nodes in the bottom row of Figure 2. Node $1L$ represents the long bursts being

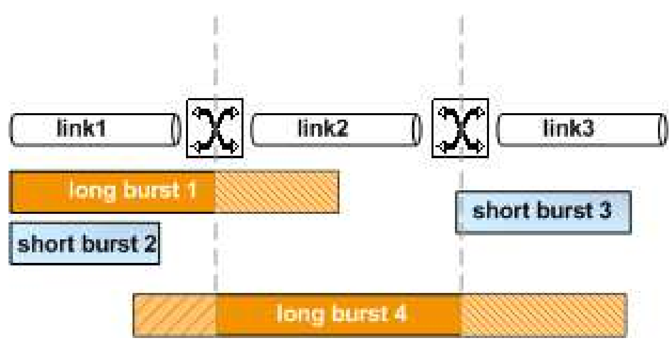

Fig. 3: Burst occupation of links

transmitted from the end-device through OXC 1 and link 1 and thus only occupying a wavelength on link 1. Upon exiting node $1L$ a long burst moves to node 12, where it simultaneously occupies a wavelength on link 1 and a wavelength on link 2. Each of the loss nodes $(i, i+1)$, $1 \leq i \leq K-1$, from the bottom row of the queueing network represents the number of long bursts simultaneously holding wavelengths on two adjacent links of the OBS path. Node KL represents the number of long bursts in transit on link K, through OXC K+1 to a destination end-devices. In other words, KL long bursts only occupy a wavelength on link K. Note in Figure 2 that there is no loss of long bursts from node $(K-1, K)$ to node KL because those bursts already occupy a wavelength on link K.

A customer in our queueing network represents either a short or a long burst. A short bursts propagates by moving through the loss nodes of the top row while the long bursts, which occupy a wavelength on two adjacent links at the same time, move from one loss node in the bottom row to the next. Due to the full wavelength conversion capability at each OBS node, a long burst may occupy one wavelength on link i and the same or different wavelength on link $(i+1)$.

Let us look at the example in Figure 3. This illustration represents a portion of the OBS path and two long bursts and two short bursts being transmitted over it. Long burst 1 occupies a wavelength on link 1 and a wavelength on link 2 while long burst 4 occupies a wavelength on link 2 and a wavelength on link 3. Short burst 2 occupies a wavelength only on link 1 and short burst 3 occupies a wavelength on link 3. Note that a burst frees up a wavelength on a link as soon as its tail departs an OXC. Despite the fact that the tail of long burst 4 is still in link 1, a new burst can enter link 1 at the same wavelength. Also note that as soon as the head of a burst enters a link then its assigned wavelength will be occupied for the duration of the burst. If we denote the number of bursts in each loss node by $n_{nodeindex}$, this particular scenario will be described by $n_{12} = 1$, $n_{23} = 1$, $n_1 = 1$ and $n_3 = 1$.

The maximum capacity of each loss node is W, because no more than W bursts can be transmitted over the same link at the same time and thus we have the following constraints:

$$\begin{aligned} n_i &\leq W \quad for \quad 1 \leq i \leq K \\ n_{i,i+1} &\leq W \quad for \quad 1 \leq i \leq K-1 \\ n_{iL} &\leq W \quad for \quad i = 1, K \end{aligned} \tag{3.1}$$

Furthermore, each physical link of the OBS path participates in three loss nodes of the queueing model. For example, link 2 is part of short node 2 and long nodes 12 and 23. Therefore, the following constraints are also true:

$$\begin{aligned} n_{1L} + n_1 + n_{12} &\leq W \\ n_{i-1,i} + n_i + n_{i,i+1} &\leq W \quad for \quad 2 \leq i \leq K-1 \\ n_{K-1,K} + n_K + n_{K,L} &\leq W \end{aligned} \tag{3.2}$$

As the bursts travel through the network, the holding time at each loss node is exponentially

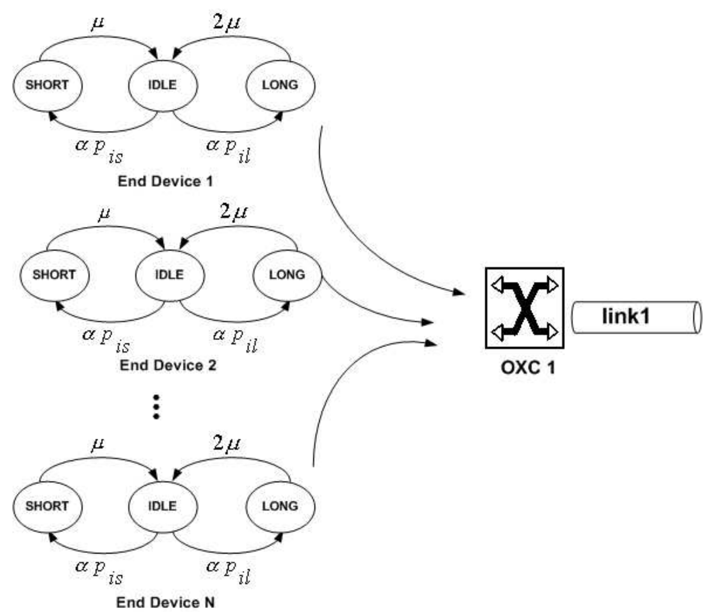

Fig. 4: Multiplexed IDLE-SHORT-LONG Sources

distributed with a mean of $1/\mu$. It is possible to model the holding time with other distributions, such as Coxian or Phase-Type, but this significantly increases the dimensionality of the state space of the queueing network.

3.2 Arrival Process

Let us consider an OBS network where each transmitting end-device is equipped with a single tunable transmitter that can transmit on any of the W wavelengths. In this case, each transmitting end-device is represented by a single traffic source. Note, however, that we can also consider the case where each transmitting end-device is equipped with W transmitters. That is, each end-device can transmit simultaneously W bursts, one per wavelength. This case can be easily modeled by simply considering $N * W$ end-devices rather than N end-devices.

An end-device which is currently transmitting a burst, has to wait until the current transmission is completely finished before it can start the transmission of the next burst. Therefore, the interarrival time between bursts transmitted by the same end-device must be dependent on the time it takes to transmit a burst. If we model the arrival process as Poisson, bursts would arrive randomly and a new burst could arrive before the transmission of the previous burst has ended. This, however, is unrealistic because the end-devices have only a single transmitter. Instead, we model the burst arrival process with an IDLE-SHORT-LONG process, which is the three-state Markov process shown in Figure 4. An OBS end-device is in the SHORT state when it is transmitting a short burst and it is in the LONG state when transmitting a long burst. It remains in a transmitting state for the duration of the burst, which is exponentially distributed. If the OBS end-device is not transmitting a burst then it is in the IDLE state, which is also exponentially distributed with a mean of $1/\alpha$. Note, that in IDLE-SHORT-LONG arrival process, the source transmits only *one* burst at a time and then it moves to the IDLE state. After that IDLE state, the probability of transmitting short burst is p_{is} and the probability of transmitting long burst is p_{il}.

The total traffic due to the N transmitting end-devices, referred to as the *cross* traffic, is a multiplexed stream of the burst arrivals from all N OBS end-devices. That is, there are N IDLE-SHORT-LONG traffic sources, which generate traffic as shown in Figure 4. We assume that all N end-devices are modeled with an identical IDLE-SHORT-LONG process. Upon transmission, a short burst will need a wavelength on link 1. A long burst will first need a wavelength only on link 1 and then it will need a wavelength from both links 1 and 2. Since upon transmission all bursts require a wavelength on link 1, even though there are N end-devices only W can transmit simultaneously at the same time because that is the capacity of link 1. If a source starts transmitting at a moment when link 1 is fully occupied it will move into the IDLE state.

In the queueing network, the value of n_1 is the number of sources currently transmitting short bursts. The value of n_{1L} represents the number of sources beginning the transmission of a long burst and thus occupying a wavelength only on link 1. The number of sources transmitting long bursts that are already occupying wavelengths both on link 1 and 2 is n_{12}. Therefore, $(n_1+n_{1L}+n_{12})$ sources are currently transmitting while the remaining $(N-n_1-n_{1L}-n_{12})$ are IDLE. At that particular moment a new burst can only arrive from the IDLE sources because the transmitting sources have to wait for the current transmission to be completed before attempting to transmit a new burst. In view of this, let us denote the arrival rate of short bursts from this multiplexed arrival stream as:

$$\lambda_1(n_{1L}, n_1, n_{12}) = (N - n_{1L} - n_1 - n_{12})\alpha p_{is}, \qquad 0 \leq n_{1L}, n_1, n_{12} \leq W \tag{3.3}$$

and similarly the arrival rate of long bursts from this multiplexed arrival stream as:

$$\lambda_{1L}(n_{1L}, n_1, n_{12}) = (N - n_{1L} - n_1 - n_{12})\alpha p_{il}, \qquad 0 \leq n_{1L}, n_1, n_{12} \leq W \tag{3.4}$$

Recall that in the considered OBS path, there are also local burst arrivals from other traffic sources in the network. We model this *local* traffic as a Poisson process in order to load the OBS path with extra short and long bursts. This traffic arrives at each intermediate link and it subsequently becomes part of the *cross* traffic. That is, a *local* traffic burst enters the OBS path at an intermediate link and it is routed toward one of the destination end-devices.

4 The Decomposition Algorithm

The queueing network, described in Section 3, is an *open loss queueing network*, which does not have a product form solution. However, this network has the Markovian property because both the duration of the bursts and their interarrival times are exponentially distributed. The underlying Markov process of this queueing network can be completely described by the $(2K+1)$-tuple $(n_1, n_2, ..., n_K, n_{1L}, n_{12}, ..., n_{K-1,K}, n_{KL})$, where the first K elements represent the short

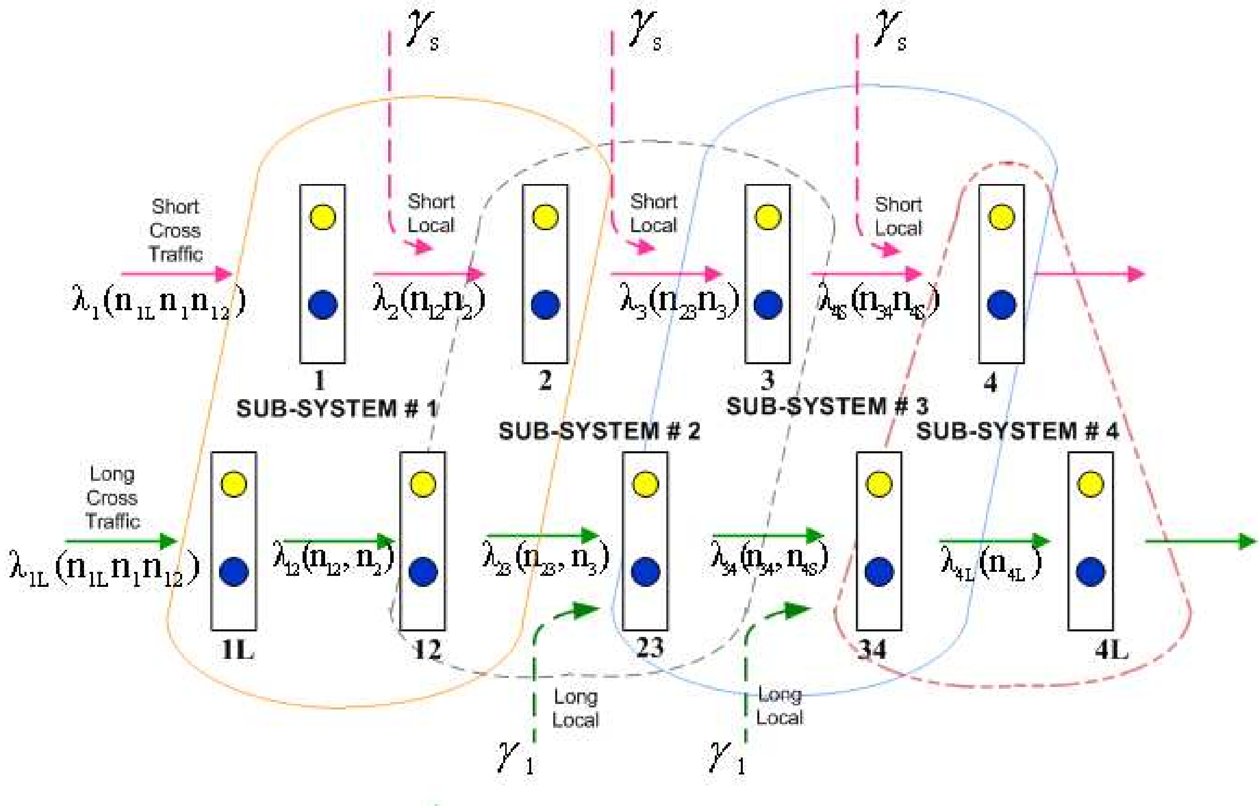

Fig. 5: Queueing Network for an OBS path of 4 links with 2 wavelengths per link

bursts in the network and the next $K+1$ elements represent the long bursts. Depending on the number of links K of the OBS path and the number of wavelengths W per link, the state space of this Markov process can become quite large. For example, an OBS path of 4 links, where each link has 8 wavelengths, will result in a Markov process with 3,905,077 states. For this reason, we analyze the network approximately, by decomposing it into small sub-systems. Each sub-system is a Markovian process and it is analyzed numerically. In order to analyze a sub-system, we need information from its adjacent sub-systems, which leads to an iterative algorithm. Below, we illustrate our decomposition algorithm through an example, and in Section 4.2 we give its more general form.

4.1 An Example

In this example, we consider an OBS path which consists of 4 WDM links. Each link has two wavelengths, i.e., $W = 2$. The cross traffic load to the OBS path is made of short bursts and long bursts and it is modeled by the multiplexed IDLE-SHORT-LONG process described in Section 3.2. In addition, there is a Poisson-distributed *local traffic* at each link and its average rate for the short bursts is γ_s and for the long bursts is γ_l.

This OBS path is modeled by the nine-node queueing network, shown in Figure 5. The queueing network is decomposed into four sub-systems. The first three sub-systems contain four nodes each while the last sub-system contains only three nodes. Sub-system 1 has nodes

1L,1,12 and 2; Sub-system 2 has nodes 12, 2, 23 and 3; Sub-system 3 has nodes 23, 3,34 and 4; Sub-system 4 has nodes 34,4 and 4L. Note that each two consecutive sub-systems *overlap* by two nodes. Sub-system 1 and sub-system 2 overlap by nodes 12 and 2, sub-system 2 and sub-system 3 overlap by nodes 23 and 3, etc. These overlap nodes are important since we use them in order to describe the interdependencies between the sub-systems.

The state of the first three sub-systems can be described by a 4-tuple vector and the fourth sub-system by a 3-tuple vector. For example, in sub-system 1 the state description vector is $(n_{1L}, n_1, n_{12}, n_2)$, where the first and third element represent the number of long bursts and the second and fourth element represent the number of short bursts. The state space of each sub-system is subject to the constraints (3.2) and (3.3). For the first three sub-systems it consists of the following states:

$$(0000), (0001), (0002), (0010), (0011), (0020), (0100), (0101), (0102), (0110), (0111), (0200), (0201),$$
$$(0202), (1000), (1001), (1002), (1010), (1011), (1100), (1101), (1102), (2000), (2001), (2002)$$

Based on the same constraints, the state space of the last sub-system is:

$$(000), (001), (002), (010), (011), (020), (100), (101), (110), (200)$$

The burst loss at each link of the OBS path is calculated from the steady-state probability vectors of each sub-system, which we denote by π_i, where $1 \leq i \leq 4$. The steady-state probabilities π_i are obtained numerically by solving the following linear equations in matrix form:

$$\pi_i Q_i = \mathbf{0} \tag{4.5}$$
$$\pi_i e_i = \mathbf{1} \tag{4.6}$$

where Q_i is the rate matrix for sub-system i and (4.6) is the normalization condition, $e_i = (1, 1, ..., 1)^T$. This system of linear equations is solved using the forward or backwards Gauss-Seidel method (see Stewart [20]).

4.1.1 Rate Matrix Generation

In order to find the steady-state probability vectors, we need to generate the rate matrix Q_i, $1 \leq i \leq 4$, for each sub-system. Below, we show in details how Q_1 is generated. The same approach is used to generate the rate matrices of the other sub-systems.

Assume that at time t, sub-system 1 is in state $(n_{1L}, n_1, n_{12}, n_2)$. Then at time $t + \triangle t$ it may transition into one of the states shown in Table 1. The rate matrix Q_1 is generated using those state transitions. Note, that in the condition for some of the transitions we need the value of n_{23}

Event	Rate	Transition States	Condition
long cross arrival to 1L	λ_{1L}	$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}+1, n_1, n_{12}, n_2)$	if $n_{1L}+n_1+n_{12} < W$
		no transition	if $n_{1L}+n_1+n_{12} = W$
short cross arrival to 1	λ_1	$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}, n_1+1, n_{12}, n_2)$	if $n_{1L}+n_1+n_{12} < W$
		no transition	if $n_{1L}+n_1+n_{12} = W$
short local arrival to 2	γ_s	$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}, n_1, n_{12}, n_2+1)$	if $n_{12}+n_2+n_{23} < W$
		no transition	if $n_{12}+n_2+n_{23} = W$
long moves from 1L to 12	$n_{1L}\mu$	$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}-1, n_1, n_{12}+1, n_2)$	if $n_{12}+n_2+n_{23} < W$
		$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}-1, n_1, n_{12}, n_2)$	if $n_{12}+n_2+n_{23} = W$
short moves from 1 to 2	$n_1\mu$	$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}, n_1-1, n_{12}, n_2+1)$	if $n_{12}+n_2+n_{23} < W$
		$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}, n_1-1, n_{12}, n_2)$	if $n_{12}+n_2+n_{23} = W$
long departure from 12	$n_{12}\mu$	$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}, n_1, n_{12}-1, n_2)$	always
short departure from 2	$n_2\mu$	$(n_{1L}, n_1, n_{12}, n_2) \rightarrow (n_{1L}, n_1, n_{12}, n_2-1)$	always

Table 1: Possible State Transitions

but in sub-system 1 we do not know its value. We need n_{23} in order to determine whether link 2 is full, i.e, $n_{12} + n_2 + n_{23} = W$. We address this problem by conditioning the state transitions in sub-system 1 on information from sub-system 2. We use nodes 12 and 2 which are common for both sub-systems 1 and 2. In sub-system 2, we use the steady-state probability π_2 to find the conditional probability r that link 2 is not full given that there are exactly n_{12} bursts in node 12 and n_2 bursts in node 2:

$$r(n_{12}, n_2) = Prob\{n_{23} < W - n_{12} - n_2 | \, n_{12}, n_2\} \tag{4.7}$$

We use this probability in sub-system 1. Instead of determining whether the equality $n_{12} + n_2 + n_{23} < W$ is satisfied, we use the probability $r(n_{12}, n_2)$. For example, the successful rate of transition for short bursts from node 1 to node 2 becomes $r(n_{12}, n_2)n_1\mu$. In other words, the sub-system transitions with that rate from state $(n_{1L}, n_1, n_{12}, n_2)$ to state $(n_{1L}, n_1 - 1, n_{12}, n_2 + 1)$. The unsuccessful rate, i.e, transition form $(n_{1L}, n_1, n_{12}, n_2)$ to $(n_{1L}, n_1 - 1, n_{12}, n_2)$ is $(1 - r(n_{12}, n_2))n_1\mu$.

4.1.2 State-Dependent Arrivals

The arrival process to sub-system 1 is given by the multiplexed IDLE-SHORT-LONG process, but still we have to determine the arrival processes to the other sub-systems. We calculate the *state-dependent arrival rate* to each of these sub-systems by using the departure rates from the previous sub-systems. These arrival rates are approximations but we found that they capture accurately the first moment of the arrival process but contain some error in the second moment.

Let us look at sub-system 2 in detail. The short bursts arrival process consists of the cross traffic, coming from node 1, and the short local traffic arriving at node 2. Note that the departure rate from node 1 is $n_1\mu$, $0 \leq n_1 \leq W$. However, in sub-system 2 we do not know the value of n_1. Therefore in sub-system 1, we express the departure rate from node 1 as a function of n_{12} and n_2 and we use that as the *state-dependent* cross arrival rate of short bursts to sub-system 2:

$$\lambda_2(n_{12}, n_2) = \sum_{i=0}^{W} Prob\{n_1 = i | n_{12}, n_2\}\, i\mu \qquad for\ 0 \leq n_{12}, n_2 \leq W \tag{4.8}$$

Similarly, the cross arrival rate of long bursts to sub-system 2 is the departure rate from node 1L, which we express as a function of n_{12} and n_2. Therefore, the *state-dependent* arrival rate of long bursts to sub-system 2 is:

$$\lambda_{12}(n_{12}, n_2) = \sum_{i=0}^{W} Prob\{n_{1L} = i | n_{12}, n_2\}\, i\mu \qquad for\ 0 \leq n_{12}, n_2 \leq W \tag{4.9}$$

Now that we have the arrival rates to each sub-system, we can generate their rate matrices and solve for their steady-state vectors.

4.1.3 The Iterative Algorithm

We analyze this queueing network with an iterative algorithm. In the initialization step, we ignore any of the link dependencies between the sub-systems in order to get initial guesses for the steady-state probabilities π_i, $1 \leq i \leq 4$. In the iterative step, we solve for π_i, $1 \leq i \leq 3$ by conditioning some of the transitions with the probabilities $r(n_{i,i+1}, n_{i+1})$, calculated based on the value of $\pi_{(i+1)}$ from the previous iteration. Next, we determine the state-dependent arrival rates for both short and long bursts to sub-system $i+1$ based on the current estimate of π_i and we solve for π_{i+1}. For the last sub-system 4, there is no need to condition so we calculate its arrival rates from π_3 and solve for π_4. We repeat the iterative step until the steady-state probabilities converge.

4.1.4 Calculation of the Burst Loss Probability

We now show how to calculate the burst loss probability for the *cross* traffic at each link of the path. This is the probability that an arriving burst at link i will find that all the wavelengths on link i are busy. We distinguish between the blocking probability of the short and the long bursts. We denote the short burst loss probability at link i with b_i. The long burst loss probability for a long burst, blocked upon moving from links $i-1$ and i to links i and $i+1$ is denoted by $b_{i,i+1}$. Note, that this long burst already has a wavelength on link i and it is blocked because link $i+1$ does not have any free wavelengths.

Since our arrivals are state-dependent, the burst loss probabilities are obtained based the steady-state vectors π in combination with the appropriate arrival rates. For example, the short burst loss probability at link 2 is given by:

$$b_2 = \frac{\sum_{i=0}^{W}\sum_{j=0}^{W} \lambda_2(i,j)\ Prob\{Link\ 2\ is\ full\ |n_{12}=i,\ n_2=j\}}{\sum_{i=0}^{W}\sum_{j=0}^{W} \lambda_2(i,j)\ Prob\{n_{12}=i,\ n_2=j\}} \tag{4.10}$$

where the probabilities are found based on π_2. The form of the numerator is justified by the fact that λ_2 is a function of n_{12} and n_2 and represents the portion of short bursts that arrive at link 2 and find all of its wavelengths busy. The denominator represents the average arrival rate of short bursts to link 2.

Similarly, we find the burst loss probability for the long bursts at link 2:

$$b_{12} = \frac{\sum_{i=0}^{W}\sum_{j=0}^{W} \lambda_{12}(i,j)\ Prob\{Link\ 2\ is\ full\ |n_{12}=i,\ n_2=j\}}{\sum_{i=0}^{W}\sum_{j=0}^{W} \lambda_{12}(i,j)\ Prob\{n_{12}=i,\ n_2=j\}} \tag{4.11}$$

where the probabilities are again found based on π_2. The form of the numerator is justified by the fact that λ_{12} is a function of n_{12} and n_2. The denominator represents the average arrival rate of long bursts to links 1 and 2.

The remaining burst loss probabilities are calculated very similarly: b_1 and b_{1L} are calculated based on π_1, b_3 and b_{23} from π_3, b_4 and b_{34} from π_4. We note that there is no burst loss at node 4L since a long burst already possesses a wavelength on link 4.

4.2 Generalization of the Algorithm

The methodology, described above, can be generalized to an OBS path with any number of links. First, construct the queueing network with two rows, one for the short bursts and one for the long bursts. In the top row, each node represents the number of short bursts that occupy a wavelength on a single link. In the bottom row each node represents the number of long bursts that simultaneously possess wavelengths on two consecutive links. Next, decompose the queueing network into sub-systems and denote the number of sub-systems by S. There are different ways to decompose the queueing network but a large number of nodes per sub-system and a large number of wavelengths will result in an unmanageable number of states. We recommend the decomposition from Section 4.1 because it contains the smallest number of nodes per sub-system and enough information to describe the interdependencies between the sub-systems. For a sub-system with 4 nodes, the number of states is a function of the number or wavelengths and it can be calculated by:

$$\sum_{l=0}^{W+1}\left[l\sum_{i=1}^{l} i\right] \tag{4.12}$$

The generalized algorithm is outlined in Algorithm 1. Note that the superscript refers to the iteration number and the convergence condition is met when the steady-state probabilities converge, i.e, $|\pi_j^{(i)} - \pi_j^{(i-1)}| < \epsilon$, $1 \leq j \leq S$ and $\epsilon = 10^{-3}$. Once the steady-state probability vectors π_j, $1 \leq j \leq S$ are computed, the burst loss probabilities at each link are obtained as described in Section 4.1.4.

```
//Initialization Step ;
for each sub-systems j = 1 : S do
    if j = 1 then
        //arrival rate of short bursts ;
        λ_1^(1)(n_1L, n_1, n_12) = (N − n_1L − n_1 − n_12) α p_is ;
        //arrival rate of long bursts ;
        λ_1L^(1)(n_1L, n_1, n_12) = (N − n_1L − n_1 − n_12) α p_il ;
    else
        calculate the state-dependent arrival rate to sub-system j based on π_{j−1}^(1);
    //ignore the link interdependencies between the sub-systems ;
    generate Q_j^(1) ;
    solve for π_j^(1) ;
//Iterative Step ;
for i = 2 until convergence do
    for each sub-systems j = 1 : S do
        if j ≠ 1 then
            calculate state-dependent arrival rates to sub-system j based on π_{j−1}^(i);
        //no link interdependencies for the last sub-subsystem ;
        if j ≠ S then
            calculate r_j^(i) based on π_{j+1}^(i−1);
        generate Q_j^(i) ;
        solve for π_j^(i) ;
```

Algorithm 1: Generalization for a Queueing Network with Any Number of Nodes

5 Numerical Results

We now present numerical results for the *cross* traffic burst loss probability at each link of an OBS path by utilizing our decomposition algorithm and we compare them to simulation results. The analytical results are obtained using Matlab code, while the simulation results are obtained with a custom event-driven C++ simulator. We show results for specific examples but we tested our algorithm for a variety of input parameters and we found it to have good accuracy.

In the simulation, we use the same arrival processes as in the analytical solution. The cross traffic is generated by a multiplexed stream of IDLE-SHORT-LONG sources. As the short bursts traverse, they occupy a wavelength on a single link of the OBS path. As the long bursts traverse they require a wavelength on two consecutive links. For each reported statistic, we ran the simulation 30 times for a sufficiently long time in order to compute the 95% confidence intervals. The simulation results are plotted along with their 95% confidence intervals but the confidence intervals are quite small and hardly visible on the plots.

For verification of our analytical solution, we present results for OBS paths with *low* and *high* traffic loads. On each figure we describe the OBS path by indicating the number of total links K and the number of wavelengths W per link. We model an OBS path of K links as a queueing network of $2K+1$ loss nodes split into two rows (see Section 3). The top row represents the short bursts and has K nodes and the bottom row is for the long bursts and has $K+1$ nodes. The input traffic load is generated by N OBS transmitting end-devices, each modeled by an IDLE-SHORT-LONG source. We vary the intensity of the traffic load based on the time spent in the IDLE state, i.e, based on the value of α. The local arrivals are Poisson-distributed and their average rate γ is the same at each link of the OBS path. For these experiments we set the mixture of bursts to 50% short and 50% long both for the cross and the local traffic.

	Loss Probabilities		**Avg. Arrival Rates**	
Link	**Short**	**Long**	**Short**	**Long**
1	14.1	14.2	1.0	1.0
2	6.1	6.1	1.3	1.6
3	5.3	6.0	0.7	0.8
4	4.4	4.8	0.2	0.4
5	3.0	3.6	0.2	0.2
6	5.9	6.7	0.2	0.3

Table 2: High Load, Relative % Error

Results for low traffic load are shown in Figures 6-8. We plot the short burst loss probability in Figure 6 and in Figure 7 we plot the long burst loss probability at each link i, for various values of α. The plots verify that our analytical results match quite closely the results obtained through simulation. The burst loss relative errors are shown in Table 2. The highest relative error at link 1 is due to the very small value of the burst loss probability at this link.

One interesting observation is that the blocking probabilities for the short and the long bursts at each link are equal, which is due to the fact that we consider an OBS network with wavelength conversion. Upon arrival at link i the short bursts release a wavelength from link $(i-1)$ and require an available wavelength on link i. Therefore, the short burst are lost if there are no available wavelengths on link i. On the other hand, the long bursts first arriving to link i are currently occupying wavelengths on links $(i-2)$ and $(i-1)$ and will require wavelengths on $(i-1)$ and i. Since the long bursts already occupy a wavelength on link $(i-1)$ they are lost only if there is no idle wavelengths on link i. Therefore, upon arrival at link i both the short and the long bursts only required a free wavelength on link i and therefore the burst loss probability for both is equal. Also, note that at low load the bursts loss probabilities gradually increase. This is due to the local traffic injected at the intermediate links and it can also be observed in

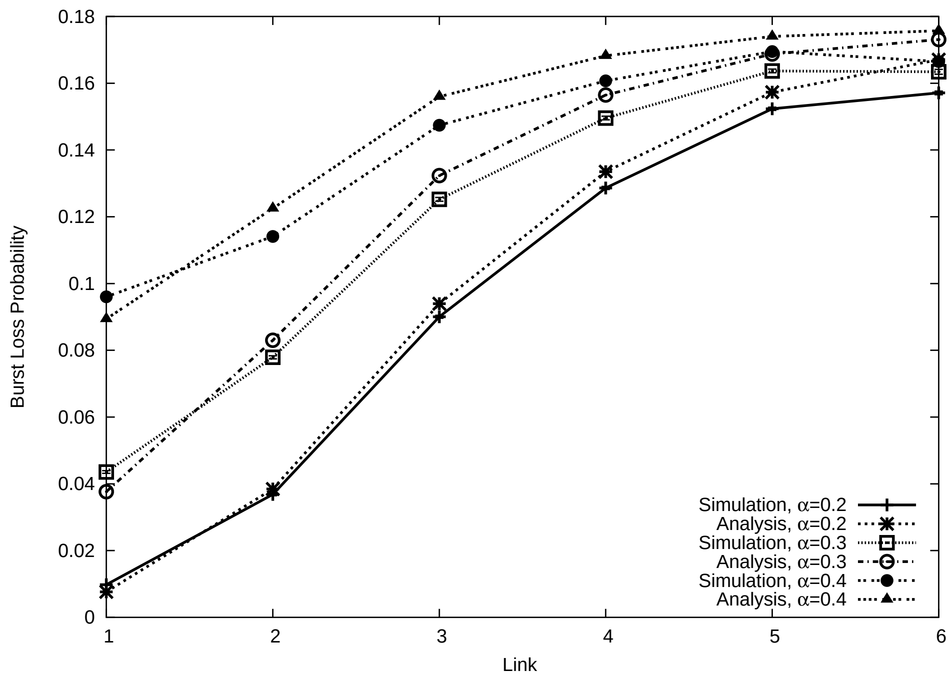

Fig. 6: Low Load, **Short** Bursts, W=8, K=6, N=16, $\gamma = 1$

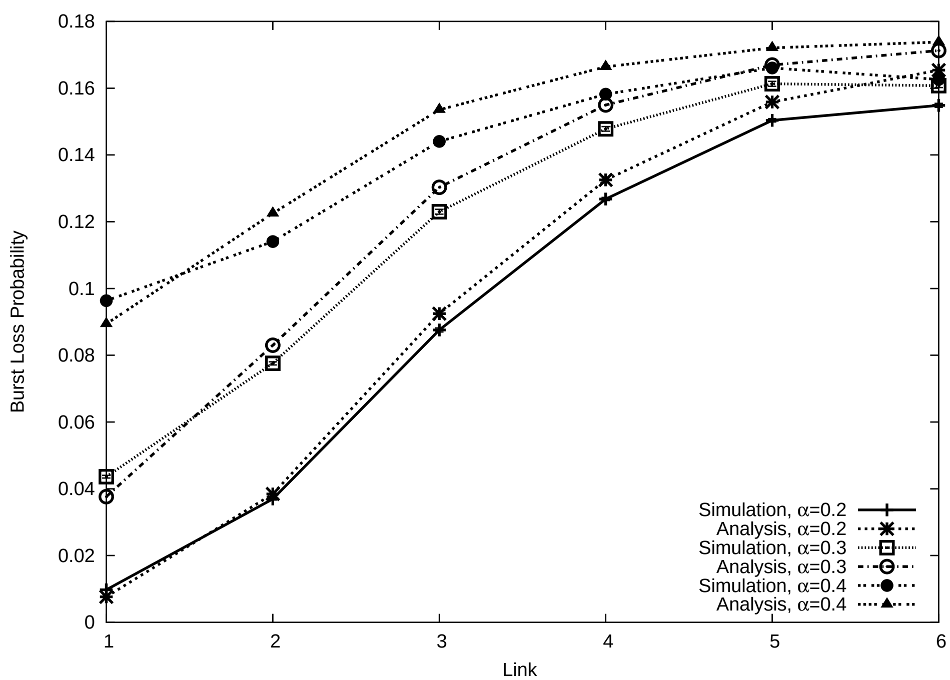

Fig. 7: Low Load, **Long** Bursts, W=8, K=6, N=16, $\gamma = 1$

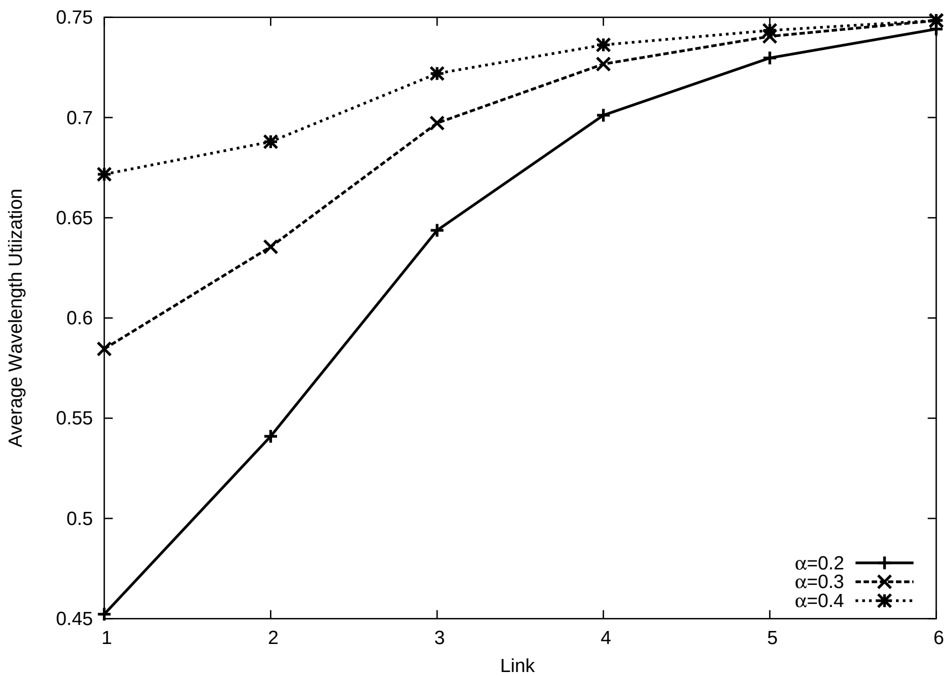

Fig. 8: Low Load, Avg. Wavelength Utilization, W=8, K=6, N=16, $\gamma = 1$

the wavelength utilization (the percentage of time that a wavelength is occupied by a burst) at each link, shown in Figure 8. The link utilization gradually increases from link 1 to link 6 and reaches 75%.

In our analytical solution, we use the approximation of the state-dependent arrival rates at each link. In Table 2 we present results to verify this approximation by showing the relative error of the average burst arrival rates at each link. The accuracy of this approximation is shown to be very good, less than 2%.

Results for high traffic load are shown in Figures 9-11. Note that even at high input load our analytical solution has a very good accuracy both for the short and the long bursts. The average relative error of the burst loss probability is less than 9% and the relative error of the average burst arrival rate is less than 4%, see Table 3.

	Loss Probabilities		**Avg. Arrival Rates**	
Link	**Short**	**Long**	**Short**	**Long**
1	9.0	8.8	3.4	3.5
2	6.0	6.2	2.9	3.4
3	4.0	4.6	2.0	2.1
4	3.5	3.6	1.4	1.6
5	2.0	2.5	1.0	1.5
6	4.6	5.6	1.3	1.7

Table 3: High Load, Relative % Error

At high traffic load, we observe a *filtering effect* of the burst loss probabilities. That is, as

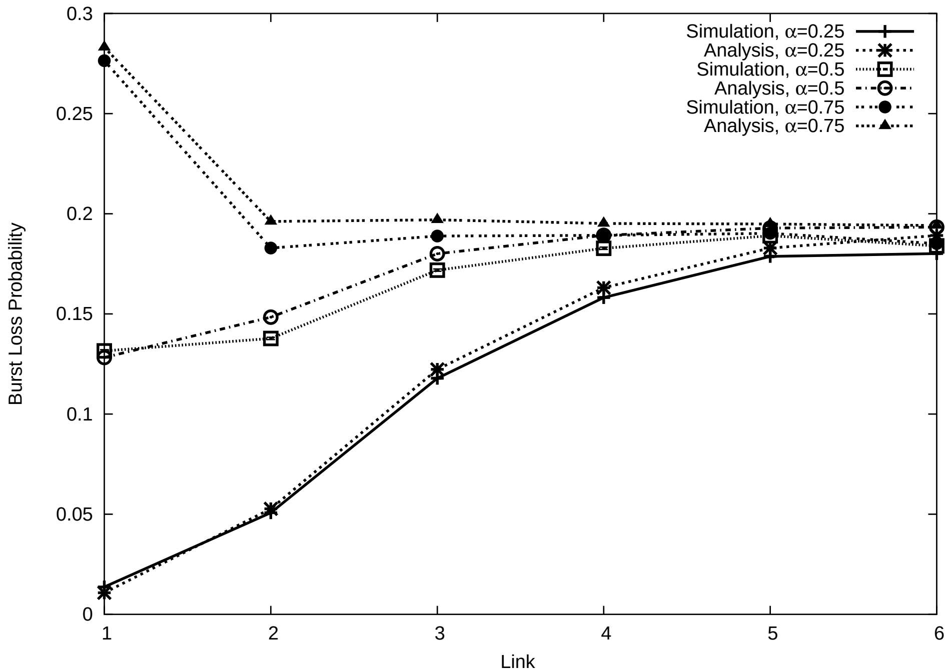

Fig. 9: High Load, **Short** Bursts, W=10, K=6, N=16, $\gamma = 1$

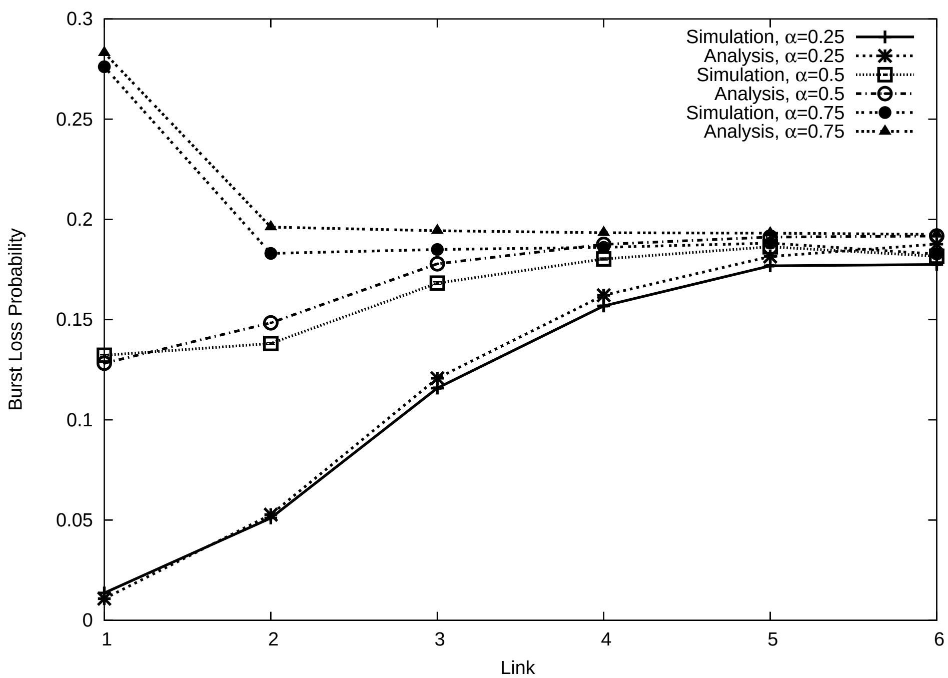

Fig. 10: High Load, **Long** Bursts, W=10, K=6, N=16, $\gamma = 1$

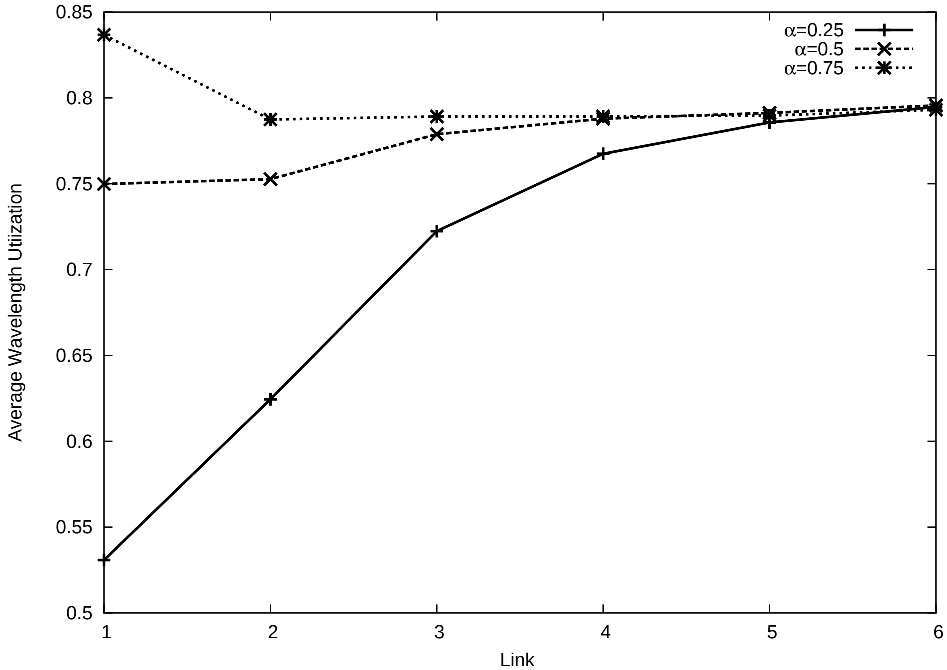

Fig. 11: High Load, Avg. Wavelength Utilization, W=10, K=6, N=16, $\gamma = 1$

we increase the load at the front of the OBS path, the burst loss probability rapidly increases but that phenomenon does not carry over to the other links in the path. Each link acts as a filter because it drops some of its incoming bursts and thus the load to the following link is reduced. In Figure 11 we show the average wavelength utilization. For these specific values, the wavelength utilization of links 4-6 converges to about 80% irregardless of the input load.

As we noted, for a given mixture of short and long bursts the blocking probabilities for both types of bursts is equal. However, increasing the number of long bursts in the network increases the overall burst loss probability. This effect is captured in Figure 12 where we plot the overall burst loss probability as we vary the percentage of short and long bursts in the network. We did this by varying the value of p_{is} in the IDLE-SHORT-LONG sources. However, the value of p_{is} effects the load of a source, i.e., the percent of time a source is busy transmitting. Therefore as we varied p_{is} we also varied α in order to keep the load of each source to be a constant in this experiment.

At $p_{is} = 0$, all the cross and the local bursts in the network are long and the burst loss is the highest for all links. As we add short bursts in the network, i.e, we increase p_{is}, the burst loss probability decreases. For this experiment, we also plotted the average utilization per wavelength in Figure 13. We observe that as the percentage of short bursts increases the utilization per wavelength decreases. This is due to the fact that long bursts occupy wavelengths on two successive links. So, each link holds the long bursts twice as long compared to the short bursts. There is a trade off between the burst loss probability and the wavelength utilization. The optimum solution is to set the mixture of short and long bursts such that the utilization is high with an acceptable bursts loss probability.

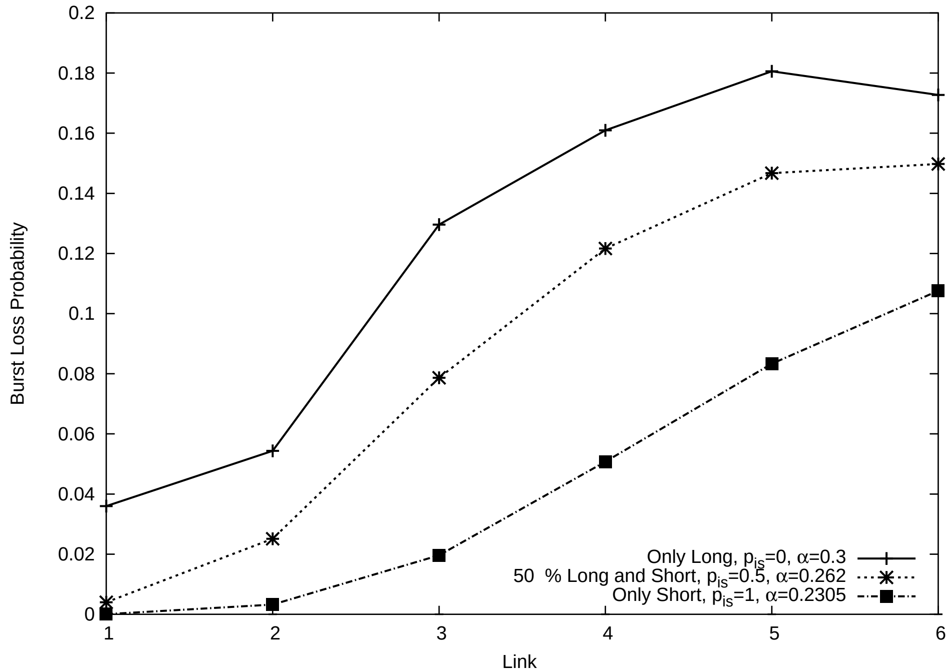

Fig. 12: Loss Probability while varying the mixture of Short and Long Bursts, W=16, K=6, N=32, $\gamma = 2$

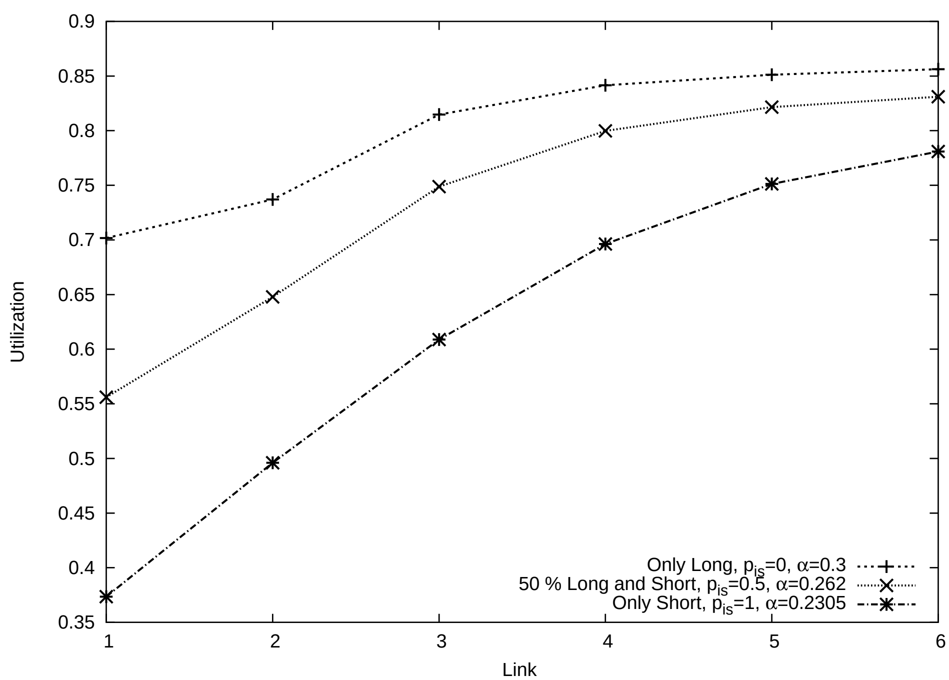

Fig. 13: Utilization while varying the mixture of Short and Long Bursts, W=16, K=6, N=32, $\gamma = 2$

6 Conclusions

We have developed an analytical model that describes the performance of an OBS path, with a mixture of short and long bursts. We use an IDLE-SHORT-LONG multiplexed arrival process, which accurately captures the burst transmission at the edge of the network. We developed a decomposition algorithm and we calculated the end-to-end burst loss probabilities at each link of the OBS path. Our algorithm was validated by comparing it to simulation results.

We found a *filtering effect* at high load traffic due to the burst losses at each link. This phenomenon can not be accurately captured in studies of a single OBS node. In addition, we found that the burst loss probability increases as the percentage of long bursts in the network increases. We also found that as the number of short bursts in the network increases the wavelength utilization decreases. Therefore, at the edge of the OBS network the ratio of short and long bursts has to be controlled in order to achieve high resource utilization while keeping the burst loss probability within an acceptable range.

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